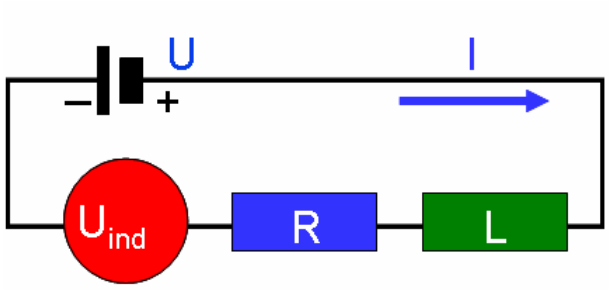
Deduction of the speed-torque behaviour of a DC motor

An alternative way for deducing the speed torque equation is to look at the motor as an electrical circuit.



Applying a fixed voltage U a steady -state current I will flow. The applied voltage has to overcome the voltage drop due to the motor inductance L, and due to the ohmic resistances R. In addition, the applied voltage has to work against the induced voltage U_{ind} which is also called back-EMF.

$$U = L \cdot \frac{\partial I}{\partial t} + R \cdot I + U_{ind}$$

For a DC motor the resulting equation can be simplified. The inductive turn cancels if we consider a steady-state current (This is what the term DC motor essentially means).

$$U = R \cdot I + U_{ind}$$

Rearranging terms we get that the induced voltage U_{ind} equals the applied motor voltage U minus the voltage drop stemming from the resistance.

$$U_{ind} = U - R \cdot I$$

Now, we replace the electrical term "induced voltage" by the mechanical term "speed" with the help of the speed constant. The same we do for current and torque with the torque constant.

$$\frac{n}{k_n} = U - R \cdot \frac{M}{k_M}$$

Multiplying the equation with k_n and observing the relationship between k_n and k_M we get the same equation between motor speed n and torque M as deduced from the power balance.

$$n = k_n \cdot U - \left(\frac{30'000}{\pi} \cdot \frac{R}{k_M^2} \right) \cdot M$$

$$n = k_n \cdot U - \frac{\Delta n}{\Delta M} \cdot M$$